

Spin-measurement induced effects in charge-related observables

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We study the dynamics of the measurement processes in a spin dependent quantum dot system, where an unsharp and a sharp detection scenario is introduced. The back-action of the unsharp detection related to the spin observables, proposed in terms of the continuous quantum measurement theory, is observed via the von Neumann measurement (sharp detection) of charges. We show that the continuous observation may accelerate the transitions between the spin states (quantum anti-Zeno effect), in contradiction with rapidly repeated observations, when it slows down the transitions. Application of the two detector system, detecting the charge and spin observables, shows a partial quantum Zeno effect in the charge current. Due to the formalism, we can calculate the fluctuation spectrum $S(\omega)$ of the charge detector, containing the Rabi frequency damping, driven by the growing unsharp detection strength γ . The Fano factor, F was also calculated as the function of γ , and shows a decreasing character, when the quantum anti-Zeno effect occurs in the charge current.

I. INTRODUCTION

Back-action of measurement on the measured object is an emblematic property of quantum systems. Quantum Zeno effect (QZE) - suppression of the object's coherent internal dynamics by measurement - is one of the marked scenarios for that back-action. In this work we are going to discuss the back-action effect in the framework of unsharp measurements, for which there is a well-developed theoretical model called continuous quantum measurement theory (cf. Ref. [1] and references therein). The unsharp measurement extracts only partial information about an observable, so we introduce another detector with sharp detection. We are interested in the output of the sharp detection, which means that the unsharp measurement will be treated in a *nonselective* picture. The *nonselective* description represents a measurement, where our record of data wasn't read out. The sharp detector also has its back-action, projecting the system randomly into its eigenstate, but now the readout will be kept and the further evolution of the system will be thrown away. The application we have in mind is the spin dependent single quantum dot, available in high quality due to massive progress in experimental technology. Quantum dots and also spin manipulations are important for the realization of qubits. The effects of spin decoherence related to the quantum computing was studied by Ref. [2]. There is a growing culture of indirect measurements on nanostructures by means of Coulomb-coupled quantum point contacts, single-electron transistors, or double quantum dot's [3]-[6]. QZE is one of the effects studied from the beginning [3] (cf. also [7]), and the concept of time-continuous measurement has penetrated the field [3-5] for a long time. Previous research has monitored the sharp or unsharp detection of the electron number. While the work presented here examines the sharp

detection of the electron number, it also contains an unsharp detection of spin observables. Spin manipulation and magnetization detection in quantum dot has been studied in experiment by Ref. [8]. The external field used for spin manipulation, can be viewed as an environment of the subsystem, the quantum dot. The whole quantum system dynamics is reversible, and tracing out the degrees of freedom of the environment we arrive to a non-unitary time evolution [9]. In general, all these non-unitary processes are connected through the Kraus-form (Appendix A) related to the completely positive mappings of the density matrices [10]. The subsystem's non-unitary dynamic imposed by the external field can be interpreted as an unsharp measurement [11]. These manipulations are time-continuous, so we will study the spin currents and magnetization detections in the framework of the continuous quantum measurement theory. This article is organized as follows. In Sec. II we introduce our model. We derive the equation of motion by the usual Born-Markov approximation and calculate the Markovian current operators. We introduce the charge detector by extending the original Hilbert-space and we derive again the dynamics and operators mentioned above. The continuous measurement theory starts from this point. The results of quantum Zeno effect are shown and discussed in Secs. III, IV. General and continuous quantum measurement theories has a wide literature and to ensure the background of this work we present a short summary of this topic in the Appendices A and B using Refs. [1, 12, 13].

II. THE MODEL

Back-action of a spin related observable is discussed by the nonselective continuous quantum measurement model (Appendix B). Measuring observable \hat{A} by a tool, which is treated quantum mechanically, we have the fol-

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lowing equation of motion [14],[15]:

$$\frac{d\hat{\rho}}{dt} = \mathcal{L}\hat{\rho}(t) - \frac{\gamma_A}{8}[\hat{A}, [\hat{A}, \hat{\rho}]], \quad (1)$$

where the main parameter of the theory, the detection performance (or detection strength), defined as

$$\gamma_A = (\Delta t)^{-1}(\Delta A)^{-2}, \quad (2)$$

where Δt is the time-resolution (or, equivalently, the inverse bandwidth) of the detector (detecting observable \hat{A}) and ΔA is the statistical error characterizing unsharp detection of the average value of the observable \hat{A} in the period Δt .

The nonselective model throws away the time dependent detection result of the observable \hat{A} , but our goal is to monitor the time dependency of the charge observable in a sharp detection scenario, by adding a charge detector to our system. Our model will contain two detectors, one is measuring continuously in the nonselective way, and the other is measuring the whole system output.

First of all let's derive our system equations without any detector. Let us consider our system, the spin dependent quantum dot, subject of experimental work [16, 17], which is coupled to two separate electron reservoirs. The density of states in the reservoirs is very high (continuum), however the dot contains only isolated levels. Taking into account the spin degrees of freedom (s), we introduce the Coulomb interaction (U), and we also add an spin flip Hamiltonian with frequency Ω . In this case the full Hamiltonian becomes

$$\hat{H} = \hat{H}_D + \hat{H}_R + \hat{H}_I, \quad (3)$$

which describes the quantum dot,

$$\begin{aligned} \hat{H}_D = & \sum_s E_s \hat{a}_{D,s}^\dagger \hat{a}_{D,s} + U \hat{a}_{D,s}^\dagger \hat{a}_{D,s} \hat{a}_{D,-s}^\dagger \hat{a}_{D,-s} \\ & + \hbar\Omega(\hat{a}_{D,s}^\dagger \hat{a}_{D,-s} + \hat{a}_{D,-s}^\dagger \hat{a}_{D,s}), \end{aligned} \quad (4)$$

the reservoirs (leads),

$$\hat{H}_R = \sum_{l,s} E_{l,s} \hat{a}_{l,s}^\dagger \hat{a}_{l,s} + \sum_{r,s} E_{r,s} \hat{a}_{r,s}^\dagger \hat{a}_{r,s}, \quad (5)$$

and the tunnel coupling between the reservoirs and dot,

$$\begin{aligned} \hat{H}_I = & \sum_{l,s} \hbar\omega_{l,s}(\hat{a}_{l,s}^\dagger \hat{a}_{D,s} + \hat{a}_{D,s}^\dagger \hat{a}_{l,s}) \\ & + \sum_{r,s} \hbar\omega_{r,s}(\hat{a}_{r,s}^\dagger \hat{a}_{D,s} + \hat{a}_{D,s}^\dagger \hat{a}_{r,s}), \end{aligned} \quad (6)$$

where $s = \pm 1/2$ and the subscripts l and r enumerate correspondingly the (very dense) levels in the left and right leads.

The state of the combined system, dot and leads, is given by the full density matrix $\hat{\chi}(t)$. The states of interest are the electronic states on the dot, described by the reduced density matrix of the dot, $\hat{\rho}(t) = Tr_R(\hat{\chi}(t))$. Here, Tr_R is the trace taken over the leads, averaging over the degrees of freedom of the reservoirs.

The usual Born-Markov master equation can be derived [9]:

$$\begin{aligned} \frac{d\hat{\rho}}{dt} = & \mathcal{L}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H}_D, \hat{\rho}(t)] \\ & - \frac{1}{\hbar^2} \int_0^\infty Tr_R \left[\hat{H}_I^R(\tau), [\hat{H}_I^D(-\tau), \hat{\rho}(t) \otimes \hat{R}_0] \right] d\tau, \end{aligned} \quad (7)$$

where

$$\begin{aligned} \hat{H}_I^R(t) = & e^{\frac{i}{\hbar}\hat{H}_R t} \hat{H}_I e^{-\frac{i}{\hbar}\hat{H}_R t}, \\ \hat{H}_I^D(-t) = & e^{-\frac{i}{\hbar}\hat{H}_D t} \hat{H}_I e^{\frac{i}{\hbar}\hat{H}_D t}. \end{aligned}$$

The \hat{R}_0 is the density matrix of the leads in thermal equilibrium. This equation of motion is for sequential tunneling, which is enough to describe our model. The cotunneling effect can be found in higher order of perturbation in the master equation approach [18].

Using the fact, that the spin flip doesn't occur under the tunneling, which means that the energy levels of the dot are much more larger than the energy of spin flip, we avoid this Hamiltonian contribution from the interaction picture. Assuming that the current flows from left to right ($\mu_l > E_s, U > \mu_r$) and the reservoirs are in thermal equilibrium the following equations can be derived:

$$\dot{\rho}_{aa} = -(\Gamma_{l,\frac{1}{2}} + \Gamma_{l,-\frac{1}{2}})\rho_{aa} + \Gamma_{r,\frac{1}{2}}\rho_{bb} + \Gamma_{r,-\frac{1}{2}}\rho_{cc} \quad (8)$$

$$\dot{\rho}_{bb} = +i\Omega(\rho_{bc} - \rho_{cb}) - (\Gamma'_{l,-\frac{1}{2}} + \Gamma_{r,\frac{1}{2}})\rho_{bb} + \Gamma_{l,\frac{1}{2}}\rho_{aa} + \Gamma'_{r,-\frac{1}{2}}\rho_{dd} \quad (9)$$

$$\dot{\rho}_{bc} = -i\delta\rho_{bc} + i\Omega(\rho_{bb} - \rho_{cc}) - \frac{\Gamma'_{l,\frac{1}{2}} + \Gamma'_{l,-\frac{1}{2}} + \Gamma_{r,\frac{1}{2}} + \Gamma_{r,-\frac{1}{2}}}{2}\rho_{bc} \quad (10)$$

$$\dot{\rho}_{cb} = i\delta\rho_{cb} - i\Omega(\rho_{bb} - \rho_{cc}) - \frac{\Gamma'_{l,\frac{1}{2}} + \Gamma'_{l,-\frac{1}{2}} + \Gamma_{r,\frac{1}{2}} + \Gamma_{r,-\frac{1}{2}}}{2}\rho_{cb} \quad (11)$$

$$\dot{\rho}_{cc} = -i\Omega(\rho_{bc} - \rho_{cb}) - (\Gamma'_{l,\frac{1}{2}} + \Gamma_{r,-\frac{1}{2}})\rho_{cc} + \Gamma_{l,-\frac{1}{2}}\rho_{aa} + \Gamma'_{r,\frac{1}{2}}\rho_{dd} \quad (12)$$

$$\dot{\rho}_{dd} = -(\Gamma'_{r,\frac{1}{2}} + \Gamma'_{r,-\frac{1}{2}})\rho_{dd} + \Gamma'_{l,-\frac{1}{2}}\rho_{bb} + \Gamma'_{l,\frac{1}{2}}\rho_{cc}, \quad (13)$$

where we used the basis $|a\rangle = |0\rangle$ for empty, $|b\rangle = |\uparrow\rangle$ for spin up ($s = \frac{1}{2}$), $|c\rangle = |\downarrow\rangle$ for spin down ($s = -\frac{1}{2}$) and $|d\rangle = |\uparrow\downarrow\rangle$ for fully occupied dot state. The left tunneling rates are

$$\Gamma_{l,s} = 2\pi \sum_l |\omega_{l,s}|^2 \delta(E_{l,s} - E_s) f_{F-D}(E_{l,s}), \quad (14)$$

$$\Gamma'_{l,s} = 2\pi \sum_l |\omega_{l,s}|^2 \delta(E_{l,s} - E_s - U) f_{F-D}(E_{l,s}),$$

and the right ones are

$$\Gamma_{r,s} = 2\pi \sum_r |\omega_{r,s}|^2 \delta(E_{r,s} - E_s) (1 - f_{F-D}(E_{r,s})), \quad (15)$$

$$\Gamma'_{r,s} = 2\pi \sum_r |\omega_{r,s}|^2 \delta(E_{r,s} - E_s - U) (1 - f_{F-D}(E_{r,s})).$$

We also introduce $\delta = (E_{\frac{1}{2}} - E_{-\frac{1}{2}})/\hbar$, the difference of the energy levels, which are renormalized by the Lamb-shifts.

The spin up and spin down current operators can be calculated from the continuity equation [19, 20]:

$$\hat{I}_s + \mathcal{L}^\dagger \hat{N}_s = 0. \quad (16)$$

This is in the Hilbert-space of the dot, but also other equivalent formulation can be used:

$$\begin{aligned} Tr_D(\hat{I}_s \hat{\rho}) = & \quad (17) \\ - \int_0^\infty Tr_{D+R} \left(\left[[\hat{Q}_s, \hat{H}_I^R(\tau)], \hat{H}_I^D(-\tau) \right] \hat{\rho} \otimes \hat{R}_0 \right) d\tau, \end{aligned}$$

Here $\hat{Q}_s = \sum_r \hat{a}_{r,s}^\dagger \hat{a}_{r,s} - \sum_l \hat{a}_{l,s}^\dagger \hat{a}_{l,s}$ is the number of electron with spin s flowed through the quantum dot.

We get the spin current operators:

$$\begin{aligned} 2\hat{I}_{\frac{1}{2}} = & \Gamma_{l,\frac{1}{2}}|a\rangle\langle a| + \Gamma_{r,\frac{1}{2}}|b\rangle\langle b| + \Gamma'_{l,\frac{1}{2}}|c\rangle\langle c| \\ & + \Gamma'_{r,\frac{1}{2}}|d\rangle\langle d|, \end{aligned} \quad (18)$$

$$\begin{aligned} 2\hat{I}_{-\frac{1}{2}} = & \Gamma_{l,-\frac{1}{2}}|a\rangle\langle a| + \Gamma'_{l,-\frac{1}{2}}|b\rangle\langle b| + \Gamma_{r,-\frac{1}{2}}|c\rangle\langle c| \\ & + \Gamma'_{r,-\frac{1}{2}}|d\rangle\langle d|. \end{aligned} \quad (19)$$

At the moment we know the dynamics of system and the observables, which we want to detect unsharply. This is enough to build up our model containing the continuous quantum measurement theory. The method is same, achieved by Ref. [14, 21], but here the application is for a different system. In the following we extend the Hilbert space adding the charge detector to the system. We will apply the continuous quantum measurement theory after the calculation of the equation of motion and the spin current operators in this new Hilbert space.

We are introducing the charge detector, making our detection results, which will describe the counting statistics with the master equation approach. The problem is that the number of tunneled particles is actually a bath and not a system observable. We perform the counting statistics by extending our Hilbert space by $|n\rangle$ which is the state of the charge detector [22]. This charge detector can be defined by two way: first type is the Ramo-Shockley one [23], where all type of tunnelings are counted, or just counting the arrived electrons in the right lead detector [22, 24]. Formally this is done by modifying the interaction Hamiltonians in the Ramo-Shockley case:

$$\hat{a}_{D,s} \hat{a}_{r,s}^\dagger \rightarrow \hat{a}_{D,s} \hat{a}_{r,s}^\dagger \otimes \hat{b}^\dagger, \quad \hat{a}_{D,s}^\dagger \hat{a}_{l,s} \rightarrow \hat{a}_{D,s}^\dagger \hat{a}_{l,s} \otimes \hat{b}^\dagger, \quad (20)$$

$$\hat{a}_{D,s}^\dagger \hat{a}_{r,s} \rightarrow \hat{a}_{D,s}^\dagger \hat{a}_{r,s} \otimes \hat{b}, \quad \hat{a}_{D,s} \hat{a}_{l,s}^\dagger \rightarrow \hat{a}_{D,s} \hat{a}_{l,s}^\dagger \otimes \hat{b}, \quad (21)$$

and for the right lead detector:

$$\hat{a}_{D,s} \hat{a}_{r,s}^\dagger \rightarrow \hat{a}_{D,s} \hat{a}_{r,s}^\dagger \otimes \hat{b}^\dagger, \quad \hat{a}_{D,s}^\dagger \hat{a}_{l,s} \rightarrow \hat{a}_{D,s}^\dagger \hat{a}_{l,s} \otimes \hat{\mathcal{I}}, \quad (22)$$

$$\hat{a}_{D,s}^\dagger \hat{a}_{r,s} \rightarrow \hat{a}_{D,s}^\dagger \hat{a}_{r,s} \otimes \hat{b}, \quad \hat{a}_{D,s} \hat{a}_{l,s}^\dagger \rightarrow \hat{a}_{D,s} \hat{a}_{l,s}^\dagger \otimes \hat{\mathcal{I}}, \quad (23)$$

where $\hat{\mathcal{I}}$ is the identity operator, and the other terms of the system Hamiltonian will be extended by the tensor-product with the identity. The charge detector excitation operator

$$\hat{b}^\dagger = \sum_{n=-\infty}^{\infty} |n+1\rangle\langle n|. \quad (24)$$

This operator increase the state if one tunneling happened in the Ramo-Shockley case or one electron is created in the right lead. Counting all possible tunnelings is

only important when the charge detection is sensitive for both leads and the excitation number has to be divide by 2 to get charge number.

After the usual Born-Markov master equation approach the new Hilbert space will be spanned by $|a\rangle \otimes |n\rangle$, $|b\rangle \otimes |n\rangle$, $|c\rangle \otimes |n\rangle$, $|d\rangle \otimes |n\rangle$. The eq. (8) is closed for states that are diagonal in n . Introduce the following representation: $\langle n|\hat{\rho}|n\rangle = \rho^n$ where ρ^n is the unnormalized conditional density matrix of the dots, depending on the number n and acting on the Hilbert space of the dot. The n -resolved density matrices are related via

$$\dot{\rho}^n \equiv L_0 \rho^n + L_+ \rho^{n-1}. \quad (25)$$

Fixing the direction of the current means that the term of ρ^{n+1} will not appear in our equations. Calculating the current operators in the new basis, we find out

$$\hat{J}_s = \hat{I}_s \otimes |n\rangle\langle n|. \quad (26)$$

We have now all the information about the system and we apply the theory of continuous quantum measurement.

The model what we will get is describing a system which is continuously detecting the spin observables without any output gained and in the same time there is a sharp detection done by a charge detector. The sharp detector is giving information about the system, and the result will depend on the interaction strength of the continuous detection.

III. RESULTS

We are going to investigate three situations, where the spin up, the spin down currents and the magnetization ($\hat{M} = \hat{a}_s^\dagger \hat{a}_s - \hat{a}_{-s}^\dagger \hat{a}_{-s}$) is unsharply detected. This operators are block diagonals in the extended Hilbert space. But also they are diagonal in the restricted space of the quantum dot. Applying the extension, which counts the electrons tunneled through the quantum dot, the eq. (1) with the continuous detection takes the form:

$$\dot{\rho}_{aa}^n = -(\Gamma_{l,\frac{1}{2}} + \Gamma_{l,-\frac{1}{2}})\rho_{aa}^n + \Gamma_{r,\frac{1}{2}}\rho_{bb}^{n-1} + \Gamma_{r,-\frac{1}{2}}\rho_{cc}^{n-1} \quad (27)$$

$$\dot{\rho}_{bb}^n = +i\Omega(\rho_{bc}^n - \rho_{cb}^n) - (\Gamma'_{l,-\frac{1}{2}} + \Gamma_{r,\frac{1}{2}})\rho_{bb}^n + \Gamma_{l,\frac{1}{2}}\rho_{aa}^n + \Gamma'_{r,-\frac{1}{2}}\rho_{dd}^{n-1} \quad (28)$$

$$\dot{\rho}_{bc}^n = -i\delta\rho_{bc}^n + i\Omega(\rho_{bb}^n - \rho_{cc}^n) - \frac{\Gamma'_{l,\frac{1}{2}} + \Gamma'_{l,-\frac{1}{2}} + \Gamma_{r,\frac{1}{2}} + \Gamma_{r,-\frac{1}{2}}}{2}\rho_{bc}^n - \Gamma\rho_{bc}^n \quad (29)$$

$$\dot{\rho}_{cb}^n = i\delta\rho_{cb}^n - i\Omega(\rho_{bb}^n - \rho_{cc}^n) - \frac{\Gamma'_{l,\frac{1}{2}} + \Gamma'_{l,-\frac{1}{2}} + \Gamma_{r,\frac{1}{2}} + \Gamma_{r,-\frac{1}{2}}}{2}\rho_{cb}^n - \Gamma\rho_{cb}^n \quad (30)$$

$$\dot{\rho}_{cc}^n = -i\Omega(\rho_{bc}^n - \rho_{cb}^n) - (\Gamma'_{l,\frac{1}{2}} + \Gamma_{r,-\frac{1}{2}})\rho_{cc}^n + \Gamma_{l,-\frac{1}{2}}\rho_{aa}^n + \Gamma'_{r,\frac{1}{2}}\rho_{dd}^{n-1} \quad (31)$$

$$\dot{\rho}_{dd}^n = -(\Gamma'_{r,\frac{1}{2}} + \Gamma'_{r,-\frac{1}{2}})\rho_{dd}^n + \Gamma'_{l,-\frac{1}{2}}\rho_{bb}^n + \Gamma'_{l,\frac{1}{2}}\rho_{cc}^n, \quad (32)$$

where Γ is

$$\hat{A} = \hat{J}_{\frac{1}{2}} : \Gamma = \frac{\gamma_{\frac{1}{2}}}{8} \frac{(\Gamma'_{l,\frac{1}{2}} - \Gamma_{r,\frac{1}{2}})^2}{4} \quad (33)$$

$$\hat{A} = \hat{J}_{-\frac{1}{2}} : \Gamma = \frac{\gamma_{-\frac{1}{2}}}{8} \frac{(\Gamma'_{l,-\frac{1}{2}} - \Gamma_{r,-\frac{1}{2}})^2}{4} \quad (34)$$

$$\hat{A} = \hat{M} \otimes \hat{I} : \Gamma = \frac{\gamma_M}{8}. \quad (35)$$

The diagonal density matrix elements $\rho_{aa}^n(t)$, $\rho_{bb}^n(t)$, $\rho_{cc}^n(t)$ and $\rho_{dd}^n(t)$ are the probabilities to find: a) no electrons; b) one electron with spin up, c) one electron with spin down, and c) two electrons inside the well, at the time t when n number of electrons tunneled through the system. Summing up the partial probabilities (states of the detector) we obtain for the total probabilities, $\rho(t) = \sum_n \rho^n(t)$, the same eqs. (8),(9),(10),(11),(12),(13) without Γ terms. Tracing out the dot degrees of freedom,

we can calculate:

$$P_n(t) = Tr\{\rho^n(t)\}, \quad (36)$$

which can be interpreted as the probability of n electrons tunneled through the whole system.

The charge current via the interpretation of right lead detection:

$$I(t) = e \frac{d}{dt} \left(\sum_n n P_n(t) \right). \quad (37)$$

If we set, that the spin up state energy is bigger then the spin down state ($E_{\frac{1}{2}} > E_{-\frac{1}{2}}$), and using the fact, that the left incoherent tunnelings are related to the electron Fermi-Dirac distribution function in eqs. (14), and the right ones to the hole F-D ($\mu_l > E_s, U > \mu_r$) in eqs. (15),

we have the following relations:

$$\begin{aligned}\Gamma_{l,-\frac{1}{2}} &> \Gamma_{l,\frac{1}{2}} > \Gamma'_{l,-\frac{1}{2}} > \Gamma'_{l,\frac{1}{2}}, \\ \Gamma'_{r,\frac{1}{2}} &> \Gamma'_{r,-\frac{1}{2}} > \Gamma_{r,\frac{1}{2}} > \Gamma_{r,-\frac{1}{2}},\end{aligned}$$

assuming that the different spin states in the reservoir are occupied in the same number. We studied a finite and a zero temperature case with assumptions for further simplicity of our results:

In the *finite temperature* case due to the Fermi-Dirac distribution function properties with the assumption $\mu_l - \mu_r = \sum_s E_s + U$ and for both leads the density of states with the tunneling rates are same, we have

$$\Gamma_1 = \Gamma_{l,-\frac{1}{2}} = \Gamma'_{r,\frac{1}{2}}, \quad \Gamma_2 = \Gamma_{l,\frac{1}{2}} = \Gamma'_{r,-\frac{1}{2}}, \quad (38)$$

$$\Gamma_3 = \Gamma'_{l,-\frac{1}{2}} = \Gamma_{r,\frac{1}{2}}, \quad \Gamma_4 = \Gamma'_{l,\frac{1}{2}} = \Gamma_{r,-\frac{1}{2}}. \quad (39)$$

In the *finite temperature* case, when $T \rightarrow 0$ other type of assumptions have to be made. In this case all the left hand side tunneling parameters are equal to each other and same is true for the right ones. Assuming that the leads are same, would result no effect induced by continuous measurement scenario for the spin currents.

$$\Gamma_l = \Gamma_{l,-\frac{1}{2}} = \Gamma_{l,\frac{1}{2}} = \Gamma'_{l,-\frac{1}{2}} = \Gamma'_{l,\frac{1}{2}}, \quad (40)$$

$$\Gamma_r = \Gamma_{r,-\frac{1}{2}} = \Gamma_{r,\frac{1}{2}} = \Gamma'_{r,-\frac{1}{2}} = \Gamma'_{r,\frac{1}{2}}. \quad (41)$$

In the finite temperature case the stationary charge current reads

$$I_\infty = e \frac{2(\Gamma_1 + \Gamma_2)(2\Gamma_4\Gamma_5\Omega^2 + \Gamma_3(2\Gamma_5\Omega^2 + \Gamma_4(4\delta^2 + \Gamma_5^2)))}{8\Gamma_3\Gamma_4\delta^2 + 2\Gamma_3\Gamma_4\Gamma_5^2 + 4\Omega^2\Gamma_3\Gamma_5 + 4\Omega^2\Gamma_4\Gamma_5 + \Gamma_1(4\Gamma_5\Omega^2 + \Gamma_3(4\delta^2 + \Gamma_5^2)) + \Gamma_2(4\Gamma_5\Omega^2 + \Gamma_4(4\delta^2 + \Gamma_5^2))}, \quad (42)$$

where we introduced for simplification $\Gamma_5 = \Gamma_3 + \Gamma_4 + \Gamma$. The above formula contains the way of quantum Zeno effect appears with growing measurement performances γ_s, γ_M (see Figs. 1, 2). Here this effect doesn't lead to a total vanish of the current [3, 14]. When $\Gamma \rightarrow \infty$ the stationary current is:

$$I_\infty = e \frac{2(\Gamma_1 + \Gamma_2)\Gamma_3\Gamma_4}{\Gamma_1\Gamma_3 + (\Gamma_2 + 2\Gamma_3)\Gamma_4}. \quad (43)$$

By inspection of the density matrix we learn that this is due to damping the coherent oscillation of the spin. Also this is the reason for the “partial” Quantum Zeno effect, because the transport of the electrons is not affected totally by the damping.

In the zero temperature case the stationary charge current reads

$$I_\infty = e \frac{2\Gamma_l\Gamma_r}{\Gamma_l + \Gamma_r}. \quad (44)$$

The result means, that the continuous measurement effects do not contribute to the stationary current. The result twice bigger than the stationary spinless single dot current, because the density of states is doubled by the spin degrees of freedom. Studying the dynamics of the density matrix it turns out that the equality of different spin related incoherent tunnelings(left or right in the same time)

$$\Gamma_{l,-\frac{1}{2}} = \Gamma_{l,\frac{1}{2}}, \quad \Gamma'_{l,-\frac{1}{2}} = \Gamma'_{l,\frac{1}{2}}, \quad (45)$$

$$\Gamma_{r,-\frac{1}{2}} = \Gamma_{r,\frac{1}{2}}, \quad \Gamma'_{r,-\frac{1}{2}} = \Gamma'_{r,\frac{1}{2}}, \quad (46)$$

leads to the cancellation of measurement induced damping mechanism. If only one of the above equalities is not

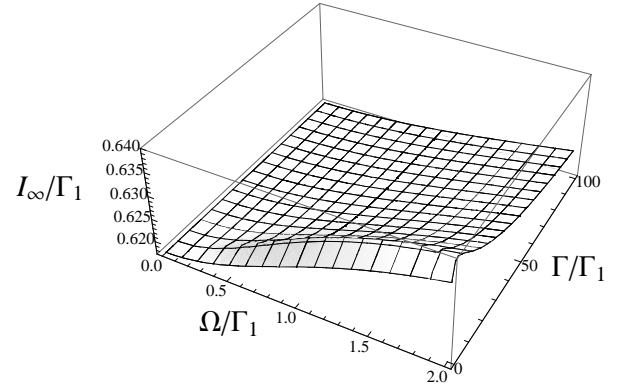


FIG. 1: Measured stationary charge current as a function of the rescaled damping and spin flip frequency parameters. $\delta/\Gamma_1 = 2$, $\Gamma_4/\Gamma_1 = 0.4$, $\Gamma_3/\Gamma_1 = 0.6$ and $\Gamma_2/\Gamma_1 = 0.8$. As the spin flip frequency(Ω) is increased, the stationary current is also increased. The increase of the measurement performance (Γ) shows the behaviour of quantum Zeno effect. This effect becomes bigger when Ω is increased. For large values of Γ a partial quantum Zeno effect can be seen.

true, the damping will survive.

Analyzing eq. (42), when the energy levels in the dot are equal ($E_{\frac{1}{2}} = E_{-\frac{1}{2}}$), then $\delta = 0$ and the reduction of the current is monotonous. For any different value of δ there is a range where the current increase with growing Γ (“anti-Zeno effect”). Also another important property is the dependence from spin flip frequency Ω : if this parameter turns to be 0, then no more measurement induced

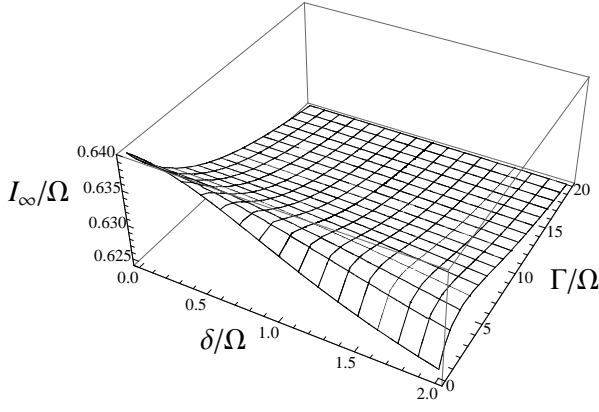


FIG. 2: Measured stationary charge current as a function of the rescaled damping and δ parameters. $\Gamma_4/\Omega = 0.4$, $\Gamma_3/\Omega = 0.6$, $\Gamma_2/\Omega = 0.8$ and $\Gamma_1/\Omega = 1$. As the difference of energy levels(δ) is increased, a quantum anti-Zeno effect can be seen.

decoherence will have effect on the system. This is also a prove how the decoherence mechanism is damping the internal coherent motion.

In a future experimental setup where will be applied this double detection system, changing the conditions of the sensitive quantum detection and analyzing the sharp detection output the validity of the continuous quantum measurement theory can be demonstrated. The substrate induced decoherence was treated, so a possible steady current decrease will only be the result of a unsharp quantum measurement. Also we have to mention that the unsharply detected operators, chosen by us, are diagonal in the Hilbert space of the dot and this is the reason why the damping mechanism has only effect on the internal coherent motion. Any other, more general operators introduce a complex damping mechanism.

Noise spectrum can be calculated by the MacDonald formula [25]:

$$S(\omega) = 2\omega \int_0^\infty \frac{d}{d\tau} \left(\langle n^2(t) \rangle - \langle n(t) \rangle^2 \right) \sin(\omega\tau) d\tau. \quad (47)$$

The Rabi oscillations, $\omega_R \approx 2\Omega$, can be recognized on Fig. 3 and due to the increase of damping mechanism induced by measurement they disappear. The peaks exact locations is also depending very strongly from the incoherent tunneling parameters. Other important property is

$$\lim_{\omega \rightarrow \infty} S(\omega) = eI_\infty. \quad (48)$$

This limit of the noise spectra is the half of the Poissonian shot noise, caused by the fermionic structure of the system.

The Fano factor by definition:

$$F = \frac{S(0)}{2eI}. \quad (49)$$

Studying Fig. 4 we realize that the Fano factor never

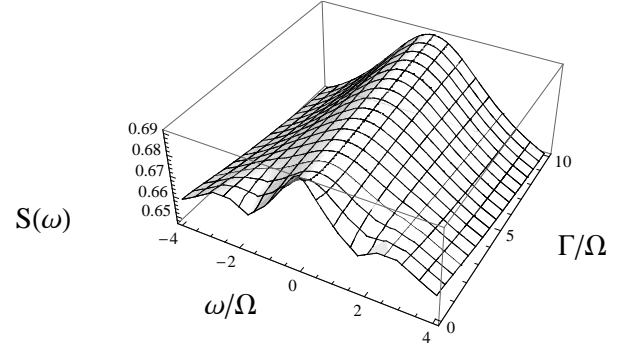


FIG. 3: Noise spectrum of the spin dependent single quantum dot as a function of Γ . $\Gamma_4/\Omega = 0.4$, $\Gamma_3/\Omega = 0.6$, $\Gamma_2/\Omega = 0.8$, $\Gamma_1/\Omega = 1$ and $\delta/\Omega = 1$. The peaks of the Rabi-frequency can be found at $\omega \approx 2\Omega$ and on increasing the performance of the measurement(Γ) their amplitudes start to decrease. For high values of Γ the Rabi peaks get overdamped and merge into a single peak around $\omega = 0$.

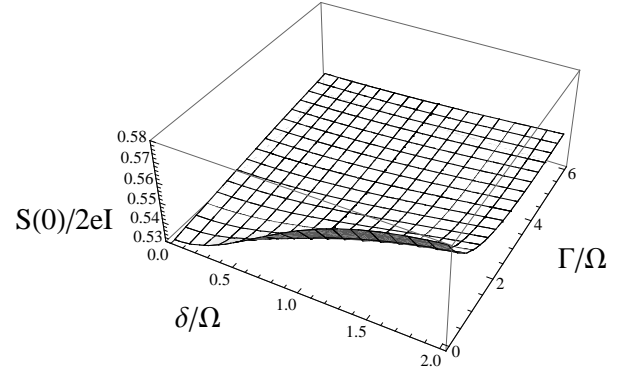


FIG. 4: The Fano factor as a function of Γ and the energy levels difference, δ . $\Gamma_4/\Omega = 0.4$, $\Gamma_3/\Omega = 0.6$, $\Gamma_2/\Omega = 0.8$ and $\Gamma_1/\Omega = 1$. As the difference of energy levels(δ) is increased, the Fano factor is also increased. Same effect can be seen, when the measurement performance(Γ) is increased. The most interesting features is for large values of δ , when the Fano factor is decreasing and after that is increasing as a function of Γ .

become Poissonian or super-Poissonian. By increasing the strength of the continuous detection an increase can be seen, but the limit of $\Gamma \rightarrow \infty$ tends to a well defined value:

$$\frac{S(0)}{2eI} = 1 - \frac{2(\Gamma_1(\Gamma_2(\Gamma_3 - \Gamma_4)^2 - 2\Gamma_3^2\Gamma_4) - 2\Gamma_2\Gamma_3\Gamma_4^2)}{(\Gamma_1\Gamma_3 + (\Gamma_2 + 2\Gamma_3)\Gamma_4)^2}. \quad (50)$$

The role of δ is similar like in the discussion of the steady current. Here, for $\delta \neq 0$, the Fano factor values are *decreasing* and after that they are increasing. The decrease can be related to the “anti-Zeno” and the increase to the quantum Zeno effect. The system studied by us shows, that the control of the energy levels(δ) and spin flip frequency(Ω) could lead to the detection of all *these*

effects in a future experiment.

IV. CONCLUSIONS

We derived an explicit expression for a double detected spin dependent single dot system. Unsharp detection of spin related observables was studied in the presence of sharp charge detection. The back-action of the unsharp detection has the character of partial quantum Zeno effect, as reducing the mean transmitted charge current at most by 5%. This effect is significant, because the charge detection is insensitive to the spin degree of freedom, but measuring the spin variable in a nonselective scenario, the spin manipulations could be detected by a charge detector. In fact, an experimental study has the potential to analyze the validity of the continuous quantum measurement theory in the mesoscopic solid state physics. In the system studied, the tunneling parameters are playing an important role because, if all the pairs of different spin tunneling parameters are equal, then there wouldn't be further effect of the backaction. The damping process of the Rabi-oscillations in the noise spectrum was observed. An other measurable quantity, the Fano factor, was studied, and it's decrease was related to the "anti-Zeno" and the increase to the quantum Zeno effect. Tight control of the coherent motion may strongly enhance the possibilities of observing the effects.

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Appendix A: General measurements

Usually the von Neumann measurement that is discussed, when the system is projected onto one of the possible eigenstates of a given observable. If we write these eigenstates as $\{|n\rangle : n = 1, \dots, n_{\max}\}$, and the state of the system is $|\psi\rangle = \sum_n c_n |n\rangle$, the probability that the system is projected onto $|n\rangle$ is $p_n = |c_n|^2$. The projectors properties are:

$$\sum_n \hat{P}_n = \hat{I}, \quad (A1)$$

$$\hat{P}_n \hat{P}_m = \delta_{n,m} \hat{P}_n. \quad (A2)$$

These sharp measurements can be discussed in two ways. *Selective* case is when the system is randomly projected into an eigenstate

$$\hat{\rho} \rightarrow \hat{\rho}_n = \frac{\hat{P}_n \hat{\rho} \hat{P}_n}{\text{Tr}[\hat{P}_n \hat{\rho} \hat{P}_n]}, \quad (A3)$$

$$\hat{A} \rightarrow A_n, \quad (A4)$$

where the possibility to detect eigenvalue A_n is $p_n = \text{Tr}[\hat{P}_n \hat{\rho} \hat{P}_n]$.

Nonselective description represent a measurement scenario, where our record of the result A_n was lost. We have a quantum system in the state $\hat{\rho}_n$ with probability p_n , but we no longer know the actual value of A_n . The state of such a quantum system is the mixture of the states $\hat{\rho}_n$ with probabilities p_n . Due to the rules of probability theory, the following result are obtained:

$$\hat{\rho} \rightarrow \hat{\rho}_n = \sum_n p_n \hat{\rho}_n = \sum_n \hat{P}_n \hat{\rho} \hat{P}_n \quad (A5)$$

$$\hat{A} \rightarrow \sum_n p_n A_n = \langle \hat{A} \rangle. \quad (A6)$$

One reason that we need to consider a larger class of measurements is so we can describe measurements that extract only partial information about an observable. A von Neumann measurement provides complete information. In this case after the measurement is performed we know exactly what the value of the observable is, since the system is projected into an eigenstate. Naturally, however, there exist many measurements, which are unable to detect sharply these eigenvalues. If the detector plus the system is under a projective measurement scenario, then the larger system will act on the system in ways that cannot be described by projective measurement on the system alone.

These *unsharp* measurements can be described by generalizing the set of projectors. The construction can be done in two way, one is where the cardinality of these sets are countable, and the other is infinite. Suppose we pick a set of N operators $\hat{\Pi}_n$, the restrictions are: $\sum_{n=1}^N \hat{\Pi}_n^\dagger \hat{\Pi}_n = \hat{I}$, where \hat{I} is the identity operator and $\hat{\Pi}_n$ to be hermitian positive semidefinite. A hermitian positive semidefinite operator can always be written as, $\hat{\Pi}_n = \hat{M}_n^\dagger \hat{M}_n$, for some operator \hat{M}_n . If the positivity of \hat{M}_n is not required, the square root of $\hat{\Pi}_n$ can give infinite solutions, which means that there are infinite different experimental apparatuses that gives the same probabilities for the outcomes.

The selective description is:

$$\hat{\rho} \rightarrow \hat{\rho}_n = \frac{\hat{M}_n \hat{\rho} \hat{M}_n^\dagger}{\text{Tr}[\hat{M}_n \hat{\rho} \hat{M}_n^\dagger]}, \quad (A7)$$

with

$$p_n = \text{Tr}[\hat{M}_n \hat{\rho} \hat{M}_n^\dagger], \quad (A8)$$

$$\sum_{n=1}^N p_n = 1, \quad (A9)$$

giving the probability of obtaining the n th outcome. The nonselective description is:

$$\hat{\rho} \rightarrow \sum_{n=1}^N p_n \hat{\rho}_n = \sum_{n=1}^N \hat{M}_n \hat{\rho} \hat{M}_n^\dagger. \quad (A10)$$

The set can be countable infinite here we have $N = \infty$ but also uncountable infinite is possible where the sum will be changed by an integral and the normalization is:

$$\int p(x)dx = 1. \quad (\text{A11})$$

These generalized measurements are often referred to as POVM's, where the acronym stands for "positive operator-valued measure." The POVM is directly refer to the set of measuring operators. The mappings of the density matrices in eqs. (A5), (A10) has a specific form, called Kraus-form [10].

Appendix B: Continuous quantum measurement of an observable

A continuous measurement is one in which information is continually extracted from a system. To construct a measurement like this, we can divide time into a sequence of intervals of length Δt , and consider an unsharp measurement in each interval. To obtain a continuous measurement, we make the strength of each measurement proportional to the time interval, and then take the limit in which the time intervals become infinitesimally short. The equation that we will derive will be valid for measurements of *any* Hermitian operator.

We now divide time into intervals of length Δt . In each time interval, we will make a measurement described by the operators

$$\hat{\Pi}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x - \hat{A})^2}{2\sigma^2}\right], \quad -\infty \leq x \leq \infty. \quad (\text{B1})$$

Each operator $\hat{\Pi}(x)$ a Gaussian-weighted sum of projectors onto the eigenstates of \hat{A} and the probability

$$p(x) = \text{Tr}(\hat{\Pi}(x)\hat{\rho}), \quad \int p(x)dx = 1. \quad (\text{B2})$$

Also true the following equation:

$$\langle x \rangle = \int xp(x)dx = \langle \hat{A} \rangle. \quad (\text{B3})$$

The strength of the continuous measurement is γ defined by

$$\gamma = \lim_{\Delta t \rightarrow 0, \sigma \rightarrow \infty} \frac{1}{\Delta t \sigma^2}. \quad (\text{B4})$$

The *selective* description such a measurement is derived by introducing x as a stochastic quantity. Main reason is to keep the randomness of the selective description and due to eq. (B3) we have:

$$x = \langle \hat{A} \rangle + \frac{1}{\sqrt{\gamma}}\xi, \quad (\text{B5})$$

where ξ is the zero-mean white-noise. A continuous measurement results if we make a sequence of these measurements and take the limit as $\Delta t \rightarrow 0$. As this limit is taken, more and more measurements are made in any finite time interval, but each is increasingly weak. We can derive this equation of motion for the system under this continuous measurement by calculating the change induced in the quantum state by the single unsharp measurement, represented by the operator $\hat{\Pi}(x)$, in the time step Δt , to first order in Δt . This measurement is performed in each time step. The procedure gives

$$|\Psi(t + \Delta t)\rangle = \frac{1}{\sqrt{p(x)}} \hat{\Pi}(x)^{1/2} |\Psi(t)\rangle \quad (\text{B6})$$

Introducing $\Delta W = \xi \Delta t$, the Wiener process, which is also a Gaussian random variable and expanding the $\hat{\Pi}(x)^{1/2}$ to first order in Δt , which gives

$$|\Psi(t + \Delta t)\rangle \propto \left[1 - \frac{\gamma}{8} (\langle \hat{A} \rangle - \hat{A})^2 \Delta t - \frac{\sqrt{\gamma}}{2} (\langle \hat{A} \rangle - \hat{A}) \Delta W \right] |\Psi(t)\rangle, \quad (\text{B7})$$

we used that in the limit of $\Delta t \rightarrow 0$, then $(\Delta W)^2 \rightarrow \Delta t$. These equation doesn't preserve the norm of the wave function, so we have to calculate $p(x)$ in the first order expansion in Δt , including the stochastic calculation rule induced above. Now we take the limit $\Delta t \rightarrow 0$, setting $\Delta t = dt$, $\Delta W = dW$ and $(\Delta W)^2 = dt$ we get

$$\begin{aligned} |\Psi(t + \Delta t)\rangle &= |\Psi(t)\rangle + d|\Psi\rangle, \\ d|\Psi\rangle &= \left[-\frac{\gamma}{8} (\langle \hat{A} \rangle - \hat{A})^2 dt - \frac{\sqrt{\gamma}}{2} (\langle \hat{A} \rangle - \hat{A}) dW \right] |\Psi\rangle. \end{aligned} \quad (\text{B8})$$

This is the equation which describes the evolution of the state of a system in a time interval dt and the observer obtains the measurement result

$$d\lambda = \langle \hat{A} \rangle dt + \frac{dW}{\sqrt{\gamma}} \quad (\text{B9})$$

in that time interval.

This is a *stochastic Schrödinger equation* (SSE) is usually described as giving the *conditioned* evolution of measurement results, and the state $|\psi\rangle$ evolves randomly. We

can also write this SSE in terms of the density operator $\hat{\rho}$ instead of $|\psi\rangle$. Using the same stochastic calculation rules, and defining $\hat{\rho}(t + dt) = \hat{\rho}(t) + d\hat{\rho}$, we have

$$d\hat{\rho} = -\frac{\gamma}{8} \left[\hat{A}, \left[\hat{A}, \hat{\rho} \right] \right] dt + \frac{\sqrt{\gamma}}{2} \left(\hat{A}\hat{\rho} + \hat{\rho}\hat{A} - 2\langle \hat{A} \rangle \hat{\rho} \right) dW \quad (\text{B10})$$

This is referred to as a *stochastic master equation* (SME).

The *nonselective* description is when the observer makes the continuous measurement, but throws away the information regarding the measurement results, the observer must average over the different possible results. Since ρ and dW are statistically independent, $\ll \rho dW \gg = 0$ (average over all possible realisations and over the Hilbert space states), and by setting to zero all terms proportional to dW in eq. (B10), we get

$$\frac{d\hat{\rho}}{dt} = -\frac{\gamma}{8} \left[\hat{A}, \left[\hat{A}, \hat{\rho} \right] \right]. \quad (\text{B11})$$

The double-commutator describes the decoherence caused by the continuous quantum measurement.

Eq. (B11) also can be achieved without introducing the random variables. For a time interval Δt the nonselective evolution of the density matrix is

$$\hat{\rho}(t + \Delta t) = \int \hat{\Pi}(x)^{1/2} \hat{\rho}(t) \hat{\Pi}(x)^{1/2} dx. \quad (\text{B12})$$

Expanding this equation into series up to the leading term Δt and calculating the integral, the following result can be obtained

$$\hat{\rho}(t + \Delta t) = \hat{\rho}(t) - \frac{\gamma}{8} \left(\hat{A}^2 \hat{\rho}(t) - 2\hat{A}\hat{\rho}(t)\hat{A} + \hat{\rho}(t)\hat{A}^2 \right) \Delta t. \quad (\text{B13})$$

Taking the limit $\Delta t \rightarrow 0$ we arrive to the same equation as eq. (B11).

Under unitary evolution, the following transformation is added to eq. (B8)

$$|\psi\rangle \rightarrow |\psi\rangle + d|\psi\rangle = \left(1 - i \frac{\hat{H}}{\hbar} dt \right) |\psi\rangle, \quad (\text{B14})$$

where \hat{H} is the Hamiltonian. For the eqs. (B10),(B11) the transformation is:

$$\hat{\rho} + d\hat{\rho} = \hat{\rho} - \frac{i}{\hbar} [\hat{H}, \hat{\rho}] dt. \quad (\text{B15})$$

If we want to treat a non-unitary dynamics then the infinitesimal transformation $\mathcal{L}\hat{\rho}dt$ can be added only to eqs. (B10),(B11). We have to remark also that the form of the equations is the consequence of the Gaussian choice of $\hat{\Pi}(x)$.

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